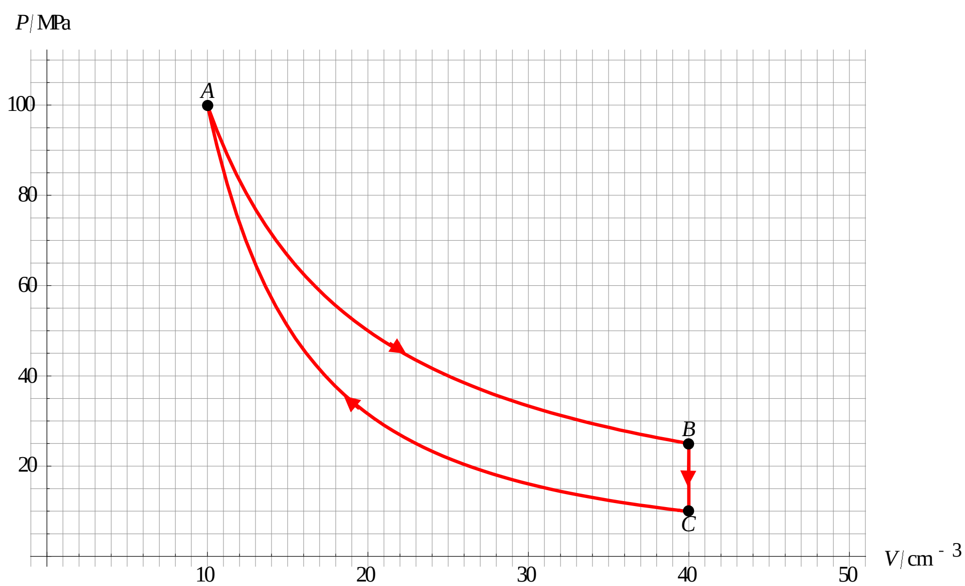
Teacher notes
Topic B

A basic problem in thermodynamics

A heat engine operates with an ideal gas in the cycle ABCA as shown. AB is an isothermal and CA is an adiabatic. The temperature at A is 580 K.



The work done by the gas along the isothermal expansion AB is 1400 J.

(a) Calculate the number of moles in the gas.

(b) Estimate the temperature at C.

(c)

(i) Explain why heat is taken out of the gas along BC.

(ii) Calculate the heat in (i).

(iii) Estimate the efficiency of the cycle.

(d) Deduce the change in internal energy of the gas along CA.

(e) Determine the entropy change along BC.

Answers

(a) At A: $n = \frac{PV}{RT} = \frac{100 \times 10^{6} \times 10 \times 10^{-6}}{8.31 \times 580} = 0.207 \approx 0.21$.

(b) Use $\frac{P_{B}}{T_{B}} = \frac{P_{C}}{T_{C}}$ i.e. $\frac{25}{580} = \frac{10}{T_{C}} \Rightarrow T_{C} = 580 \times \frac{10}{25} = 232 \approx 230\text{ K}$.

(c)

(i) $Q_{BC} = \Delta U_{BC} + W_{BC} = \Delta U_{BC} + 0$. (No work is being along an isovolumetric change.) $\Delta U_{BC} < 0$ because temperature decreases so $Q_{BC} < 0$

(ii) $Q_{BC} = \frac{3}{2} R n \Delta T = \frac{3}{2} \times 8.31 \times 0.207 \times (580 - 232) = 898 \approx 900\text{ J}$.

OR

$Q_{BC} = \frac{3}{2} V \Delta P = \frac{3}{2} \times 40 \times 10^{-6} \times (25 - 10) \times 10^{6} = 900\text{ J}$.

(The tiny discrepancy between the 2 answers is due to significant figures. In the first answer we would get 900 J if we used *n* = 0.207477489.)

(iii) Along AB, $\Delta U = 0$ so $Q_{AB} = 0 + W_{AB} = 1400\text{ J} = Q_{in}$. Hence,

$\eta = \frac{W_{net}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = \frac{1400 - 900}{1400} = \frac{500}{1400} = 0.36$.

(d) Along CA, $Q = 0 = \Delta U_{CA} + W_{CA}$. The net work is 500 J and so $W_{net} = 500 = W_{AB} - |W_{CA}| \Rightarrow |W_{CA}| = 1400 - 500 = 900\text{ J}$. CA is a compression so $W_{CA} = -900\text{ J}$. Hence $\Delta U_{CA} = -W_{CA} = +900\text{ J}$.

OR, much better:

$\Delta U_{cycle} = 0 = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0 - 900 + \Delta U_{CA} \Rightarrow \Delta U_{CA} = +900\text{ J}$.

(e) $\Delta S_{cycle} = 0 = \Delta S_{AB} + \Delta S_{BC} + \Delta S_{CA} = \frac{1400}{580} + \Delta S_{BC} + 0 \Rightarrow \Delta S_{BC} = -\frac{1400}{580} = -2.4\text{ J K}^{-1}$. (No heat is exchanged along the adiabatic and so $\Delta S_{CA} = 0$.)